A simplified model for meteorological now-casting

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ABSTRACT

A simplified numerical model for simulation of the interaction amongst atmospheric eddies at synoptic scale is presented and discussed . Since for all such interactions the atmosphere can be regarded as a thin layer of non-viscous, incompressible fluid, we started from a review of the bi-dimensional flows hydrodynamics in the Hamiltonian formulation to derive the dynamics for "a vorticity segments system" in motion on a spherical surface. The extension to the case of a rotating sphere becomes possible through the equation of potential vorticity conservation $d\,(\,\Gamma + f\,/h\,)/\,dt$ (where h is the effective thickness of the atmosphere and $f = 2\ \Omega\ \sin\phi$ is the Coriolis factor) which describes the inertial effects and the possible consequences of thickness variation in the atmosphere. Thus, the developed flux model allows to simulate the short range evolution of cyclonic and anticyclonic systems, within affordable computational requirements. A few simulations are presented in order to illustrate the theoretical developments and to demonstrate the procedure.

1 INTRODUCTION

In the last decades many numerical models have been devised and applied in order to forecast the weather dynamics, both on the large scale (General Circulation Models, GCM's) and on the subcontinental scale (Limited Area Models, LAM's). Recently some reliable algorithm has been checked to forecast the convection driven weather time evolution on a very small spatial scale (note that both the GCM's and the LAM's use the hydrostatic approximation to calculate the dynamics of a stratified atmosphere). Satellite images in various wave ranges are often used, together with the traditional data from the meteorological network, to control and almost continuously tune the weather forecast produced by the models (although it's still difficult to use satellite data from a quantitative point of view…). Anyway, both the GCM's and the LAM's require a suitably dimensioned computing power and take advantage from the knowledge of many experimental data in order to give their maximum performance.

On the other side, since the time when the first satellite images have been of common experience, the temptation grew to subjectively predict the position of clouds and meteorological fronts (for a maximum period of a day ahead) on the basis of the actual and some previous meteorological images mainly in the visible and infrared spectrum. In fact , in the case of not too intricate meteorological patterns, this subjective now-casting turns out to be same what successful and allow to estimate the cloud system position with errors of the order of about 100 Km in the next 12-24 hours.

Thus, a simple and fast algorithm has been devised to numerically perform this simplified form of now-casting. The method lies on the basis of a two-dimensional vortex fluid dynamics, through the pointing out of ideal centers of circulations which interact determining their overall motion following some suitably modified vortex system dynamic rules. At each instant the discrete set of the positions of the vortices determines the whole dynamic flux, which in turn determines the future motion of the vortices. This simplified scheme allows suitable short-time dynamic forecasts.

2 TWO DIMENSIONAL VORTEX DYNAMICS ON THE PLANE

It is well known that a z independent (i.e. two-dimensional and plane) velocity field $\mathbf{v}(x.y,t)$ for an incompressible, not viscous flow satisfies the dynamic system

$$\begin{cases} \partial_t \Gamma + \mathbf{v} \cdot \nabla \Gamma = 0 \\ \begin{cases} \nabla \cdot \mathbf{v} = 0 \\ \dot{\nabla} \cdot \mathbf{v} = \Gamma \end{cases} \end{cases} \tag{1}$$

where $\nabla \equiv (\partial_x, \partial_y)$, $\dot{\nabla} \equiv (-\partial_y, \partial_x)$ and Γ is the only not zero component of vorticity (along z).

Equations (1) can be solved if the initial vorticity field ($\Gamma(x,y,0)$)and suitable boundary conditions ($\mathbf{n} \cdot \mathbf{v}(x,y,t) = 0$ on each boundary point) are given. The whole problem consists in an instantaneous problem (inner brace in eqns.(1)), similar to the magnetostatic problem

$$\begin{cases} \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu \, \mathbf{j} \end{cases} \tag{2}$$

and an evolution equation for the voticity field $\Gamma(x,y,t)$, which represent the lagrangian invariance for the vorticity ($d\Gamma / dt = 0$, i.e. $\Gamma = const$ along the trajectories). Usually one drops out the continuity equation by imposing $\mathbf{v} = -\dot{\nabla}\Psi$: then the second instantaneous equation in eqns.(1) becomes a Poisson equation $\nabla^2 \Psi = -\Gamma$ for the stream function $\Psi(x,y,t)$.

2.1 Governing equations

If the initial vorticity field is given by a set of parallel straight circulation lines normal to the xy plane, with circulations values γ_i and intersection points $\mathbf{R}_i(0) = \{X_i(0), Y_i(0)\}$ with the plane (i.e. $\Gamma(\mathbf{r},0) = \sum_i \gamma_i \delta(\mathbf{r} - \mathbf{R}_i(0))$, then one can easily demonstrate that, for $t > 0$, the ' Biot-Savart ' field structure holds:

$$\Gamma(\mathbf{r},t) = \sum_i \gamma_i \delta(\mathbf{r} - \mathbf{R}_i(t)) \tag{3}$$

$$\Psi(\mathbf{r},t) = -\frac{1}{2\pi} \sum_i \gamma_i \ln|\mathbf{r} - \mathbf{R}_i(t)| \tag{4}$$

where

$$\frac{d\mathbf{R}_i}{dt} = \frac{1}{2\pi} \sum_{j \neq i} \gamma_j \frac{(\mathbf{R}_i - \mathbf{R}_j)'}{|\mathbf{R}_i - \mathbf{R}_j|^2} \tag{5}$$

or, explicitly,

$$\begin{cases} \dfrac{dX_i}{dt} = \dfrac{1}{2\pi} \sum\limits_{j \neq i} \gamma_i \dfrac{Y_i - Y_j}{(X_i - X_j)^2 + (Y_i - Y_j)^2} \\ \dfrac{dY_i}{dt} = \dfrac{1}{2\pi} \sum\limits_{j \neq i} \gamma_i \dfrac{X_j - X_i}{(X_i - X_j)^2 + (Y_i - Y_j)^2} \end{cases} \tag{6}$$

The problem can also be formulated in terms of a *Pseudo*-Hamiltonian system like the one bellow:

$$\begin{cases} \gamma_i \dfrac{dX_i}{dt} = \dfrac{\partial H}{\partial Y_i} \\ \gamma_i \dfrac{dY_i}{dt} = -\dfrac{\partial H}{\partial X_i} \end{cases} \qquad i = 1,\ldots,N \tag{7}$$

where

$$H = -\frac{1}{2\pi}\sum_i \sum_{j \neq i} \gamma_i \gamma_j \ln\left|\mathbf{R_i} - \mathbf{R_j}\right| \tag{8}$$

is the *pseudo*-Hamiltonian of the system. Thus, almost in the case of unbounded fluid, the fluid dynamic problem of eqns. (1) can be solved (in $\Re^2$) by solving the discrete nonlinear dynamic system represented by eqns.(6). if the fluid domain is bounded by geometrically simple boundaries, one can try to directly solve the problem by adding suitably defined circulations images to the r.h.s. of eqns.(6).

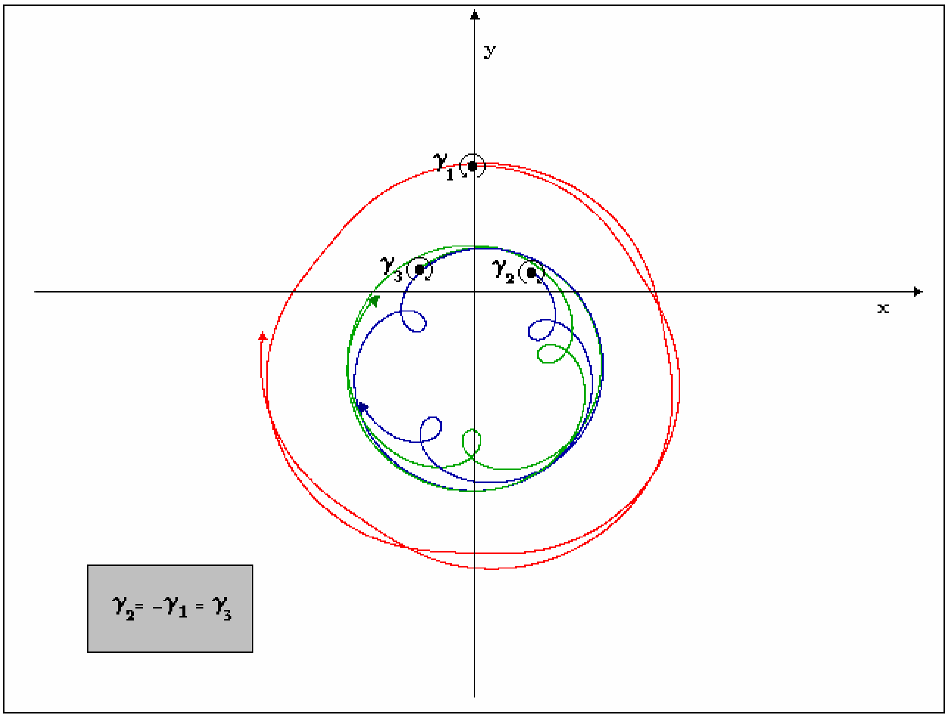

Figure 1: trajectories for a system of tree vortices with the same circulation intensity but different spin.

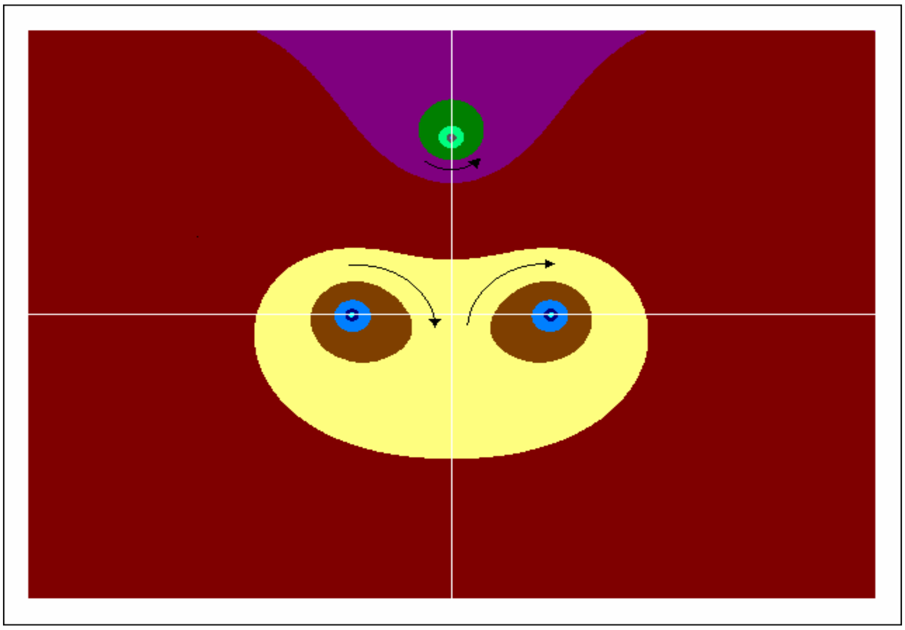

Figure 2: stream function for a system of tree vortices with the same circulation intensity but different spin

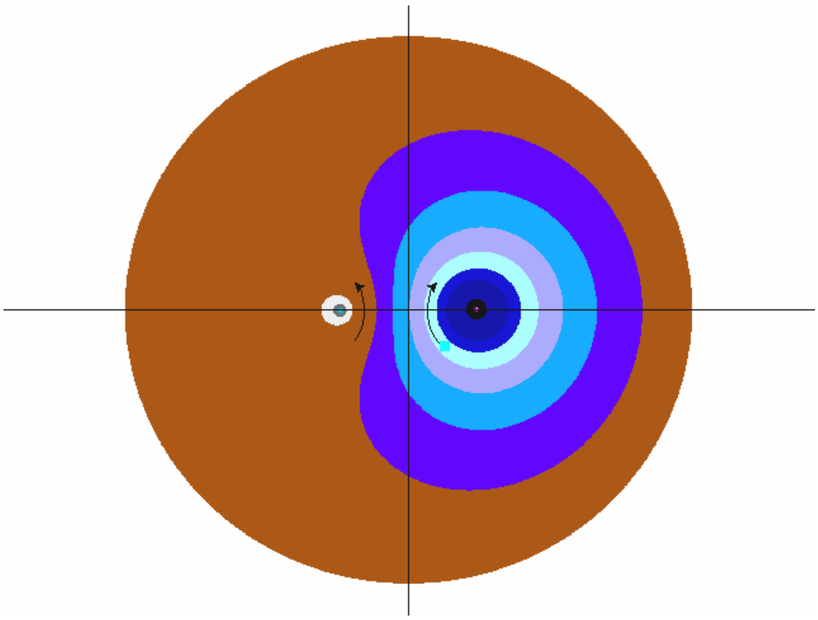

Figure 4: Stream function for a system of two vortices with different circulation intensity and spin in a circularly bounded plane.

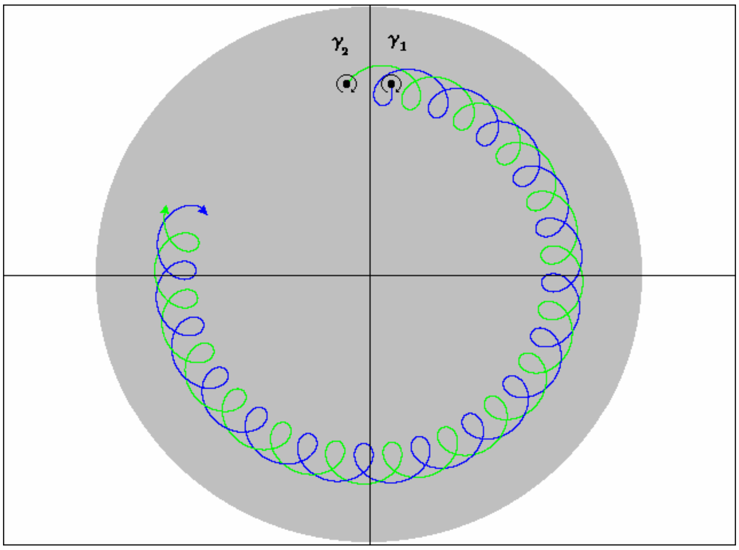

Figure 4: Trajectories for two vortices with the same circulation in intensity and spin near the circular boundary.

In figures 1 - 4 we show some trajectories for both unbounded ($\Re^2$) and circularly bounded circulation points motions and the corresponding instantaneous stream functions. As we can see from these figures also apparently simple systems (i.e. three vortices at the vertices of a equilateral triangle)can have a complex evolution. When the fluid domain is bounded in the *z* direction by two parallel planes, the *z* translation invariance of the vorticity field breaks down. Given a system of parallel vorticity segments, all initially normal to the *xy* plane, each of these segments tends in general to change its rectilinear shape into a curved one.

Nevertheless, if the distance *h* between the planes is small in comparison with the distance among the circulation segments, the velocity field is approximately independent of *z* within the fluid slab, and one can consider approximately 'rigid' parallel vorticity segments for the whole (not too long) concerned time interval. Of course, the Biot - Savart like interaction law breaks down too, and must be replaced by the elementary Biot-Savart law. This is equivalent to consider vorticity segments instead of infinite vorticity lines.

$$\frac{d\mathbf{R}_i}{dt} = \frac{1}{2\pi}\sum_{j\neq i} h\gamma_j \frac{\left(\mathbf{R}_i - \mathbf{R}_j\right)'}{\left|\mathbf{R}_i - \mathbf{R}_j\right|^3} \tag{9}$$

Thus the interactions among vortex segments vanish with their reciprocal distance more rapidly than in the *z* unbounded case.

3 FLOW INSIDE A SPHERICAL FLUID FILM

If we imagine the atmosphere as a thin ($h \cong 10$ km) spherical film surrounding the Earth (mean radius $a \cong 6378$ km),and consider inviscid, incompressible flows therein on the large synoptic scale, we are compelled to express the equations of the motions for the 'rigid' vortex elements in term of their geographical coordinates (λ_i , ϕ_i). By calculating the distance among vortex segments along the spherical chords joining them, one obtains with some algebra that the stream function for the atmospheric flow is given by

$$\Psi(\lambda,\phi,t)=\frac{h}{4\sqrt{2}\,a^3}\sum_i\frac{\gamma_i}{\left[1-\cos\phi\cos\phi_i(t)\cos(\lambda-\lambda_i(t))-\operatorname{sen}\phi\operatorname{sen}\phi_i(t)\right]^{\frac{1}{2}}} \tag{10}$$

where

$$\begin{cases}\dfrac{d\lambda_i}{dt}=\dfrac{h}{8\sqrt{2}\,a^3}\displaystyle\sum_{j\neq i}\gamma_j\,\frac{\operatorname{sen}\phi_j-\cos\phi_j\,\tan\phi_i\,\cos(\lambda_i-\lambda_j)}{\left[1-\cos\phi_j\,\cos\phi_i\,\cos(\lambda_i-\lambda_j)-\operatorname{sen}\phi_j\,\operatorname{sen}\phi_i\right]^{\frac{3}{2}}}\\ \dfrac{d\phi_i}{dt}=\dfrac{h}{8\sqrt{2}\,a^3}\displaystyle\sum_{j\neq ii}\gamma_j\,\frac{\cos\phi_j\,sin(\lambda_i-\lambda_j)}{\left[1-\cos\phi_j\,\cos\phi_i\,\cos(\lambda_i-\lambda_j)-\operatorname{sen}\phi_j\,\operatorname{sen}\phi_i\right]^{\frac{3}{2}}}\end{cases} \tag{11}$$

The angular zonal and meridional velocities of the flow, $\dot{\lambda}$ and $\dot{\phi}$ can then be calculated using the relations

$$\begin{cases}\dot{\lambda}=\dfrac{1}{\cos\phi}\dfrac{\partial\Psi}{\partial\phi}\\ \dot{\phi}=-\dfrac{1}{\cos\phi}\dfrac{\partial\Psi}{\partial\lambda}\end{cases} \tag{12}$$

In the following figures (figs. 6 - 10) we report (in stereographic projection) some trajectories for a few vorticity segments.

4 VORTEX MOTION IN A ROTATING SPHERICAL FLUID FILM

In order to represent the inertial effects which arise from observing the atmospheric flow from a rotating reference frame fixed with the Earth, we must modify the vortex motion equations considering the effects of the Earth rotation both on the velocity components and on the vortices spins.

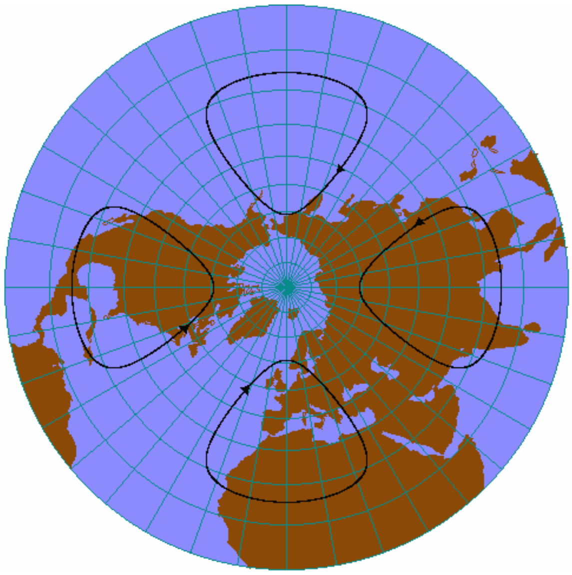

Figure 5:Trajectories for four vortices with the same circulation intensity but different spin on the sphere.

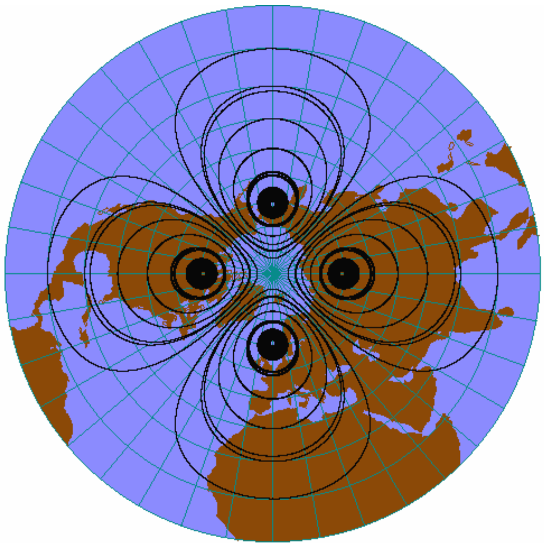

Figure 6: Stream lines for four vortices with the same circulation intensity an different spin.

As well known:

$$\mathbf{v_a} = \mathbf{v_{rel}} + \mathbf{\Omega} \times \mathbf{r} \qquad (13)$$

This equation states simply that the absolute velocity of an object on the rotating Earth is equal to its velocity relative to the Earth plus the velocity owing to the rotation of the Earth. In terms of the angular zonal and the meridional components of the velocity field the eqns.(13) is the same as adding to the zonal component the Earth angular velocity , and so we have that the eqns.(11) simply become:

$$\begin{cases} \dfrac{d\lambda_i}{dt} = \left\{ \dfrac{h}{8\sqrt{2}\,a^3} \displaystyle\sum_{j \neq i} \gamma_j \dfrac{\text{sen}\phi_j - \cos\phi_j \,\tan\phi_i \,\cos(\lambda_i - \lambda_j)}{\left[1 - \cos\phi_j \,\cos\phi_i \,\cos(\lambda_i - \lambda_j) - \text{sen}\phi_j \,\text{sen}\phi_i\right]^{\frac{3}{2}}} \right\} + \Omega \\ \dfrac{d\phi_i}{dt} = \dfrac{h}{8\sqrt{2}\,a^3} \displaystyle\sum_{j \neq ii} \gamma_j \dfrac{\cos\phi_j \, sin(\lambda_i - \lambda_j)}{\left[1 - \cos\phi_j \,\cos\phi_i \,\cos(\lambda_i - \lambda_j) - \text{sen}\phi_j \,\text{sen}\phi_i\right]^{\frac{3}{2}}} \end{cases} \qquad (14)$$

At the same time, also the absolute vorticity of the eddies is affected by the Earth rotation according to the following equation:

$$\Gamma_\mathbf{a} = \Gamma_\mathbf{rel} + f \qquad (15)$$

where $f = 2\ \Omega \sin\phi$ is the Coriolis factor. Then also the vortices circulations change with the latitude and must be recalculated at every step. Besides , we need to estimate vortex spin variation due to the thickness reduction or the increase of the fluid slab . Then , in order to estimate the spin variations we apply to the vorticity segments the potential vorticity conservation low:

$$\frac{d}{dt}\left(\frac{\Gamma + f}{h}\right) = 0 \qquad (16)$$

a simulation is presented in the next page in order to illustrate the Earth rotation effects on the vortices trajectories.

5 THE STRUCTURE OF THE WHOLE NOWCASTING PROCEDURE

In the previous sections we have briefly described a simplified theoretical model which allow the calculation of the time evolution of an incompressible, inviscid and almost bi-dimensional flow field in a thin spherical fluid shell, starting from a discrete set of well localized 'circulation points' .In order to apply this model to the atmosphere one must be able to pointing out a number of initial circulations centers from the meteorological maps or from the satellite images. Although the few example reported refer to data deduced from weather

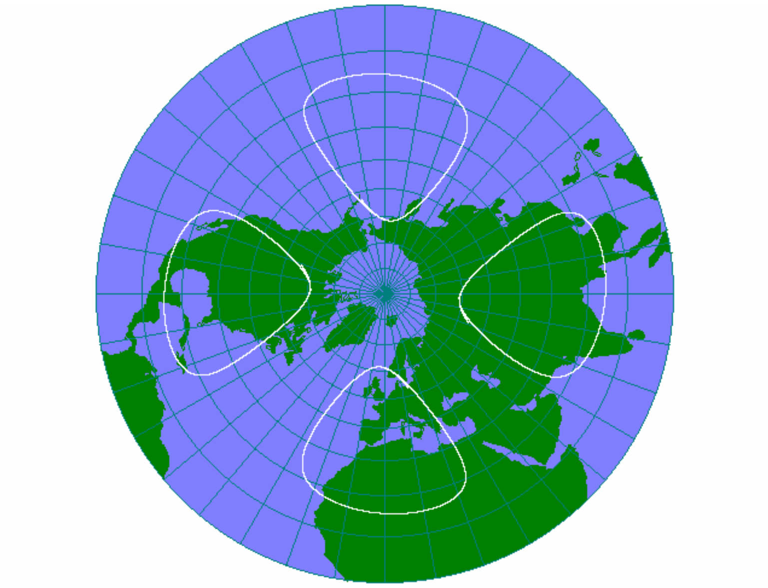

Figure 7:Trajectories for four vortices with the same circulation intensity but different spin on the rotating sphere.

charts, programs exist which are able to guess a velocity field by examining a few subsequent satellite images (usually in the infrared spectrum) following the displacements of cloud systems ; these programs use simple and fast pattern recognition algorithms in order to identify in the subsequent images the same cloud patterns even if they show forms and dimensions (slowly) varying in time during their motion. From the bi-dimensional velocity field , given on a suitable defined pixel mesh, a very simple numerical procedure can easily calculate the correspondent vorticity field and squeeze the maximum absolute vorticity domains to identify a number of ideal circulation centers and the associated circulation values. Finally, the previously described fluid dynamic model operates to produce a guess on the future velocity fields at choose time intervals. In the following figure we report, for sake of clearness, the analogous guessed displacements obtained starting from meteorological maps , comparing the guessed circulation point displacements with those observed (and reported) on subsequent meteorological charts.

6 Conclusions

The whole work has been centered on the developing of a simplified model describing the interactions between atmospheric structures in terms of vortex dynamics and able to forecast the motion of these structures at synoptic scale and on short time scales within affordable computational requirements.

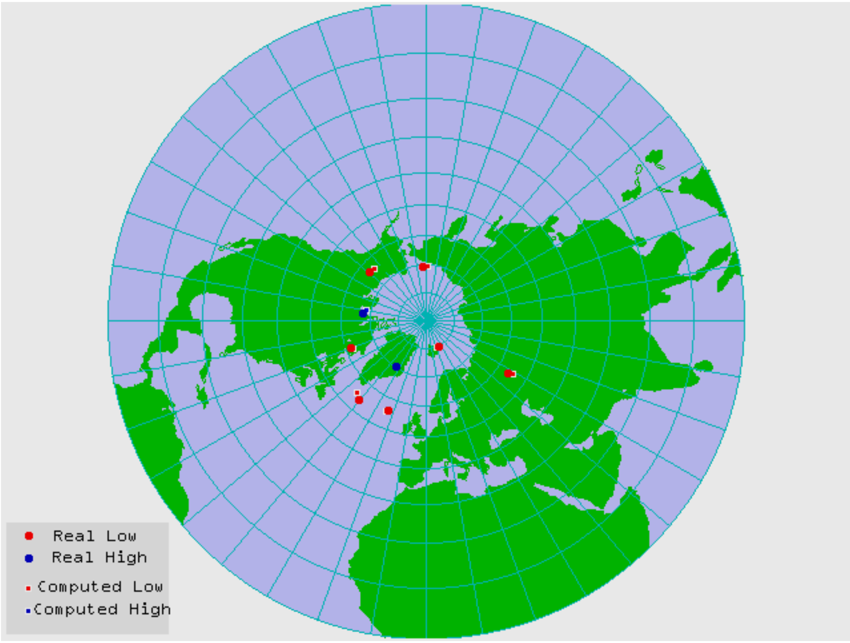

Figure 8: Simulation concerning the forecast for 07/07/1994 (forecast interval =24 h) at 500 mb and comparison between the output of the model and real situation.

Since in our hypothesis the atmosphere can be regarded as a thin layer of inviscid incompressible fluid , we started with a review of bi-dimensional flows in the Hamiltonian formulation and we considered the atmospheric vortices like localized point vortices. Before we analyzed the unbounded case and then we passed , through the hydrodynamics images method , to the case of a system of vortices that interact inside of a fluid domain delimited by a circular boundary. However , some of the approximation in this first step of the work turned out not much effective. In fact , to consider the atmospheric vortices like infinite vorticity straight lines, neglecting in this way the (geometrical) finish of the atmospheric layer, produces interaction between the vortices much stronger than the real ones. Besides the dynamics of a fluid tied up to flow on a rotating spherical surface is difficulty approximated with the one of a plane rotating fluid. Starting from these observations we have re-expressed the theory of vortices interactions by the terms of vorticity segments on a spherical surface.

For the rotating sphere case the conservation low of potential vorticity (16) describes in a effective and concise way the inertial effects and the possible effects of thickness variation in the atmospheric slab on the vortex spin.

The model , reformed by these terms is able to perform simulations of wind field evolution on short times and at synoptic scale in good agreement with the real situation pointed out from meteorological maps .

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